Non-regular Languages   Lecture - 13   1

14/05/2020

Pigeon-hole principle

Suppose there are n objects and m boxes, where $n > m$.

If all n objects are placed into m boxes, then atleast one box has more than one object.

⇓ used in Pumping Lemma

⇓ used to

1. ~~used to~~ prove that certain languages are non-regular.
2. check whether a language accepted by a FA is finite or infinite

Pumping Lemma (for Regular Languages)

- Let $M = (Q, \Sigma, \delta, q_0, F)$ be a FA.
- It has n number of states.
- Let L be the language accepted by M and assume the language is regular.
- Let $x \in L$, and $|x| \geq n$ i.e, length of string x is greater than the number of states of FA.
- If string x can be decomposed into combination of strings

$$x = uvw$$

such that

$|uv| \leq n$, $|v| \geq 1$, then

$uv^iw \in L$ for $i = 0, 1, 2, \ldots$

Proof :- – Let $M = (Q, \Sigma, \delta, q_0, F)$ be a FA.

– Let $x = a_1 a_2 \ldots a_m$ be an input accepted by the machine.

– Assume the machine has $\underline{n}$ states and $m \geq n$.

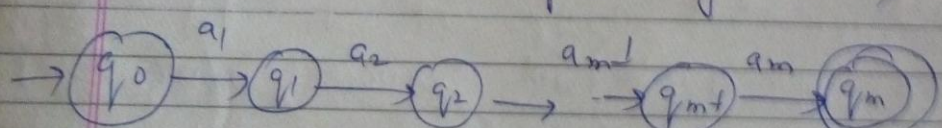
Note :-> i) If exactly one symbol is accepted by a FA, then there are two distinct states in it.

ii) IIIly to accept 2 symbols, FA should have 3 states.

iii) In general, to accept a string x, where $|x| = n$, FA should have $n+1$ states.

Let these states be $q_0, q_1, q_2 \ldots q_n$.

– We have m input symbols $\Rightarrow$ $m+1$ states



i.e, $\delta(q_{i-1}, a_i) = q_i$ for each $1 \leq i \leq m$

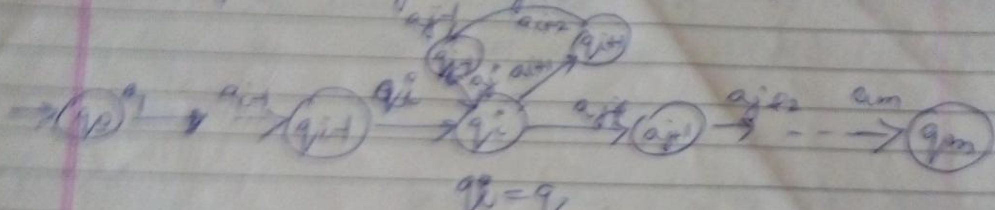
– In this chain of states from q_0 to q_m, suppose a state q appears more than once $q = q_i$ and $q = q_j$ $i < j$

In this case, the chain should have a loop and we can split this into 03 groups:

(1) The first group is the string prefix from $a_1 a_2 \ldots a_i$

(2) second group is the loop string from $a_{i+1} a_{i+2} \ldots a_{j-1} a_j$

(3) third group is the string suffix from $a_{j+1} a_{j+2} \ldots a_m$.



$q_i = q_j$

prefix(u) loop(v) suffix(w)

<u>DFA for Pumping Lemma</u>

- The m/c cannot remember the previous state or it does not know how it reached the current state.

— q_i and q_j are same

$$\delta(q_0, a_1 a_2 \ldots a_{i-1} a_i a_{j+1} \ldots a_m)$$

$$= \delta(\delta(q_0, a_1 a_2 \ldots a_{i-1} a_i), a_{j+1} a_{j+2} \ldots a_m)$$

$$= \delta(q, a_{j+1} a_{j+2} \ldots a_m)$$

$$= \delta(q_k, a_{k+1} a_{j+2} \ldots a_m)$$

$$= q_m$$

Applications of PL

- All languages are not regular
- A non-regular language is ~~that~~ one for which a FA cannot be constructed

(1) To prove that certain languages are not regular.

(2) to check whether the language is infinite. If there is a string x such that $|x| \geq$ the number of states accepted by the DFA, M, then $L(M)$ is infinite. Otherwise LM is finite.

Procedure (to prove that L is not regular).

(1) Assume L is regular and the no. of states in FA be n.

(2) Select the string x and break it into substrings u, v and w, such that $x = uvw$ with the constraint.

$|x| \geq n \quad |uv| \leq n$ and $|v| \geq 1$.

(3) find any i such that $uvw \notin L$, according to PL $uv^iw \in L$.

So the result is a contradiction to the assumption that L is regular.

$\Rightarrow$ given language is not regular

Q1) Show that $L = \{ww^R \mid w \in (0+1)^*\}$ is not regular (5)

Sol ① Let L be regular and n the number of states in FA.

Consider the string

$$x = \overbrace{\underbrace{1\cdots1}_{n}\underbrace{0\cdots0}_{n}}^{w}\overbrace{\underbrace{0\cdots0}_{n}\underbrace{1\cdots1}_{n}}^{w^R}$$

where

n is the number of states of FA.

$w = 1\cdots10\cdots0$

$w^R = 0\cdots01\cdots1$

② Since $|x| \geq n$, we can split the string x into uvw such that $|uv| \leq n$, $|v| \geq 1$

$$x = \underbrace{1\cdots1}_{u}\ \underbrace{\ }_{v}\ \underbrace{0\cdots0\ 0\cdots0\ 1\cdots1}_{w}$$

$|u| = n-1 \quad |v| = 1 \Rightarrow |uv| = |u| + |v|$

$= n-1+1 = n.$

which is true.

According to PL, $uv^iw \in L$ for $i = 0, 1, 2 \cdots$

③ If $i = 0$ i.e, v does not appear

$\Rightarrow$ # of 1's on the left of x is less than # of 1's in the right of x

$\Rightarrow$ string is not of form ww^R.

$\therefore uv^iw \notin L \longrightarrow$ contradiction to the assumption that L is regular.

...now that $L = \{a^n b^n \mid n \geq 0\}$ is not regular. (6)

Sol

Step ① Let L be regular and n be the # of states in FA.

② Consider $x = a^n b^n$

$|x| = 2n$ and $2n > n$.

$x = uvw \quad |uv| \leq n \quad |v| \geq 1$

$$x = \overbrace{\underbrace{aaaaaa}_{u}\,\underbrace{a}_{v}}^{n}\ \overbrace{\underbrace{bbbbbbb}_{w}}^{n}$$

$|u| = n-1 \quad |v| = 1 \Rightarrow |uv| = |u| + |v|$

$= n-1+1$

$= n$.

According to PL

$uv^iw \in L \quad i = 0, 1, 2, \ldots$

③ If i is 0, v does not appear

$\Rightarrow$ # of a's < # of b's

$\Rightarrow$ x does not contain same number of a's and b's. (a's < b's)

$i = 2, 3 \ldots$

a's will be more than b's.

$\therefore L = \{a^n b^n \mid n \geq 0\}$ is not Regular